

DESIGN OF A SINE-WAVE GENERATOR

BY

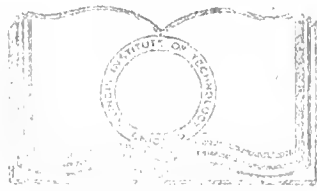
J. E. ANDERSON

ARMOUR INSTITUTE OF TECHNOLOGY

1917

537.83

An 2



ALL OF ACQUISITION
OF TECHNOLOGY
UNIVERSITY LIBRARIES

AT 432

Anderson, J. E.

Design of a true sine-wave
generator

DESIGN OF A TRUE SINE-WAVE GENERATOR

A THESIS

PRESENTED BY

JOEL E. ANDERSON

TO THE

PRESIDENT AND FACULTY

OF

ARMOUR INSTITUTE OF TECHNOLOGY

FOR THE DEGREE OF

BACHELOR OF SCIENCE

IN

ELECTRICAL ENGINEERING

MAY 31, 1917

APPROVED:

E. H. Freeman
Professor of Electrical Engineering

Dean of Engineering Studies

Dean of Cultural Studies

ILLINOIS INSTITUTE OF TECHNOLOGY
PAUL V. GALVIN LIBRARY
35 WEST 33RD STREET
CHICAGO, IL 60616



Design of a Sine Wave Generator

Table of Contents.

	Page
List of Plates	2
Object	3
Part I - Theory of Wave Form.	6
The Fundamental Equation	6
Effect of Grouping	8
Pitch Factor	11
Spread Factor	14
Design of Winding	23
Flux Distribution	26
Armature Reaction	28
Effect of Teeth	36
Part II - Calculations and Design.	40
Armature Coils	40
Binding Wires	50
Field Winding	52
Pole Shoe	56
Conclusions	70
Appendix - Bibliography	71

Design of a Sine Wave Generator

List of Plates.

Plate I - Details of Sine Wave Alternator

In pocket.

Plate II - Armature Coils In pocket.

Plate III - Pole Shoe In pocket.

Plate IV - Armature Cross Section

In pocket.

Design of a Sine Wave Generator.

Object

The investigation described in this report was undertaken with the hope of designing an alternating current generator which would give a pure sine wave of e.m.f. A generator giving a perfect sine wave shape is highly desirable because practically all specifications involving alternating current tests are based upon a sine wave form. This is especially true of the iron loss tests on transformers. All alternating current equations and theory are based upon the assumption that the e.m.f. and current vary with time according to the formula

$$i = I_m \sin(\omega t + \theta)$$

Hence in experiments with alternating currents involving the use of alternating current equations and theory the source of e.m.f. must have a perfect sine wave form. In making dielectric tests of insulators

Design of a Sine Wave Generator

using alternating current, the value of the maximum e.m.f. is desired. If a sine wave of e.m.f. is used this maximum value can easily be calculated but if a non-sine wave is used it is necessary to obtain the shape of the wave, which is quite a laborious process.

The design, which is described in the latter part of this report, was made for an old ten-pole alternating-current generator which has been in the laboratory of the Armour Institute for a number of years. This generator was a single-phase, 133-cycle, 25 KW, machine, and had a concentrated winding. The field contained a series winding which was brought out from the armature thru a commutating rectifier. In the new design this series winding was removed and additional turns were added to the direct-current field winding. The new design was made for a frequency of 60-cycles and hence the speed had

Design of a Sine Wave Generator

to be reduced from 1596 r.p.m. to 720
r.p.m.

Design of A Sine Wave Generator

Part I - Theory of Wave Form.

The Fundamental Equation.

The electro-motive force induced in a coil is dependent upon the rate of change of flux threading the coil. Let ϕ_a be the flux per pole and p the number of poles. Then one armature conductor will cut $\phi_a p$ lines of force per revolution, or

$$\phi_a p \times \frac{\text{r.p.m.}}{60} \text{ lines per second}$$

The average e.m.f. in one conductor in volts will then be

$$\phi_a p \times \frac{\text{r.p.m.}}{60} \times 10^{-8}$$

The form factor of an e.m.f. wave is the ratio of the effective voltage to the average voltage. For a sine wave of e.m.f. this value is

$$\frac{\gamma \frac{1}{2} E_{\max}}{\frac{2}{\pi} E_{\max}} = 1.11$$

Design of a Sine Wave Generator

Then the effective e.m.f. per conductor is

$$1.11 \phi_a p \times \frac{\text{r.p.m.}}{60} \times 10^{-8}$$

Since f , the frequency, = $\frac{p \times \text{r.p.m.}}{120}$, the

volts induced in each conductor will be

$$E = 2.22 \phi_a f 10^{-8}$$

For Z conductors

$$E = 2.22 \phi_a f 10^{-8} \text{ volts.}$$

This formula, however, does not take account of the grouping of the conductors, the spread factor, f_w , and the pitch of the coils or chord factor, f_c . In order to take account of these factors the equation may be changed to read

$$E = 2.22 Z \phi_a f 10^{-8} \times f_c \times f_w.$$

This expression gives the voltages induced in each belt of conductors. When two belts are connected in series the e.m.f.s add vectorially at the phase angle of the two belts.

Design of a Sine Wave Generator

Grouping.

If two belts of conductors are spaced 60 electrical degrees apart on the armature of an alternating current generator and are connected together in series, the e.m.f.s developed in these belts will add vectorially at 60 or at 120 degrees. The positions of these belts of conductors for a two-pole generator are shown at A and B in Fig. 1. The developed winding of the coils showing the way they would be connected so as to have the current in the same direction in the adjacent belts, is shown in Fig. 2. The belt e.m.f.s would thus add vectorially at 60 degrees as shown in Fig. 3 and the resultant e.m.f., OC, would be equal to $\sqrt{3}$ times the e.m.f. of either belt.

For the third harmonics in the e.m.f., the vector addition would be at three times

Design of a Sine Wave Generator

60 degrees or 180 degrees. Hence the third harmonic e.m.f.s would be directly opposed as shown in Fig. 4. As these e.m.f.s are equal in value they would exactly neutralize each other and hence would not appear in the terminal e.m.f. In the same way the ninth harmonics would add at 9 times 60 degrees or 540 degrees. This is equivalent to adding at 180 degrees and so the ninth harmonics would be cancelled out. In general, all multiples of the third harmonic will be cancelled out by connecting the coils in this way.

Design of a Sine Wave Generator

Pitch Factor.

If the pitch or throw of a coil is less than a pole pitch, or 180 electrical degrees, the e.m.f.s developed in these conductors will not be in phase but will add vectorially at an angle depending upon the number of degrees which the throw of the coil lacks of being full pitch or 180 degrees. If a coil has a throw of $\frac{4}{5}$ full pitch or 144 degrees the fundamental e.m.f.s in the two sides of the coil will be out of phase 180-144 or 36 degrees. The position of the two sides of such a coil on the armature of an alternator is shown in Fig. 5a and the developed winding is shown in Fig. 5b. The fundamental e.m.f.s will add vectorially at 36 degrees as shown in Fig. 6. Vectors OM and ON are the e.m.f.s in the two sides of the coils and OP is their resultant. OP is equal to $2 \cos 18^\circ$ times either OM or ON. The fifth

Design of a Sine Wave Generator

harmonics in the e.m.f.s in the coil will add together at 5 times 36° or 180° . Thus the fifth harmonic e.m.f.s in one side of the coil would be in opposite phase to those in the other side of the coil and as they are equal in magnitude they would cancel out and there could be no fifth harmonic in the e.m.f. across the coil terminals. If the coils were connected in series there could be no fifth harmonics or any multiples of the fifth across the machine terminals.

Design of a Sine Wave Generator

Spread Factor.

In a distributed winding, that is, where the conductors are placed side by side and connected in series to make up a belt, the e.m.f.s induced in each conductor are not in phase but vary according to the spacing between the centers of the conductors. In Fig. 1 the eight conductors in belt A cover 60° and hence the spacing between the centers of the conductors is $7\frac{1}{2}$ degrees. The e.m.f. in each conductor will, therefore, be $7\frac{1}{2}$ degrees out of phase with the e.m.f. in the adjacent conductor and the e.m.f. across the terminals of the belt will be the vector sum of the e.m.f.s in the individual conductors. If the belt covered $\frac{2}{7}$ of the pole pitch, it would take up $51\frac{3}{7}$ degrees. The phase difference between the conductors in such a belt would be equal to $51\frac{3}{7}^\circ$ divided by the number of conductors. If there ^{were} _^ nine conduct-

Design of a Sine Wave Generator

ors these e.m.f.s would add at $5 \frac{5}{7}$ degrees, as shown in Fig. 7 a. The resultant for the fundamental will be equal to 0S. For the 7th harmonic, the angle between each vector will be 7 times $5 \frac{5}{7}$ or 40 degrees, and the resultant of the vectors for the 9 conductors will be zero as shown in Fig. 7b. Hence there can be no seventh harmonic or any multiple of the seventh harmonic in the terminal e.m.f. of a belt of conductors which covers $51 \frac{3}{7}$ electrical degrees.

In general, the value of any harmonic which would be present in the flux distribution, would be equal to the number of conductors in 180° of the winding times the e.m.f. of this harmonic in one conductor, times the spread factor and pitch factor for this harmonic, times the group factor. Thus in the above case of 9 conductors in a belt of $\frac{2}{7}$ the pole pitch and with $\frac{4}{5}$ pitch,

Design of a Sine Wave Generator

the equation for the 11th harmonic, which would be the lowest numbered harmonic above the first that could appear, would be

$$18 e_{11} \times (\text{spread factor})_{11} \times (\text{pitch factor})_{11} \times (\text{group factor})_{11}$$

where e_{11} is the e.m.f. for the 11th harmonic in one conductor and the subscripts refer to the order of the harmonic. As a belt takes up $51 \frac{3}{7}$ degrees for the fundamental it would take up $11 \times 51 \frac{3}{7}$ degrees or $565 \frac{5}{7}$ degrees for the harmonic. Since there are 9 conductors the vectors for these conductors would add at $565 \frac{5}{7} \div 9$ or 62.86 degrees. This vector addition is shown in Fig. 7(c). o-a, o-b, o-c, etc. are the vectors for the 11th harmonic in each conductor. The resultant of the vector addition is the vector o-m and the spread factor is the ratio of o-m to the arithmetical sum of the individual vectors which is 9 times o-a. The value

Design of a Sine Wave Generator

of o-m is equal to

$$\begin{aligned} & 2r \sin \angle tsm \\ & = 2r \sin 77.14^\circ \end{aligned}$$

where r is the radius of the circle.

The value of the vector o-a is equal to

$$\begin{aligned} & 2r \sin \angle asu \\ & = 2r \sin 31.43^\circ \end{aligned}$$

Then the spread factor will equal

$$\begin{aligned} & \frac{2r \sin 77.14^\circ}{9 \times 2r \sin 31.43^\circ} \\ & = \frac{2 \times 0.975}{18 \times 0.5215} = 0.2077. \end{aligned}$$

The effect of the 4/5 pitch is to cause the fundamental e.m.f.s to add at 36° . This would cause the 11th harmonic e.m.f.s to add at 11×36 or 396 degrees, which is equivalent to adding at $396-360$ or 36 degrees. Hence the pitch factor for the 11th harmonic would be the same as for the fundamental or 0.951. The effect of the grouping

Design of a Sine Wave Generator

of the coils is to cause the fundamental e.m.f.s to add at 60 degrees. The 11th harmonic e.m.f.s would add at 11×60 or 660 degrees, which is equivalent to adding at $720-660$ or 60 degrees. The group factor for the 11th harmonic will therefore also be the same as for the fundamental or will be 1.732.

Hence the value of the 11th harmonic e.m.f. in 360 electrical degrees of the winding will be

$$\begin{aligned} 18 e_{11} \times 0.2077 \times 0.951 \times 1.732 \\ = 6.16 e_{11}. \end{aligned}$$

In the same way the equation for the 13th harmonic is

$$18 e_{13} \times (\text{spread factor})_{13} \times (\text{pitch-factor})_{13} \times (\text{group factor})_{13}.$$

A belt will take up $13 \times 51 \frac{3}{7}$ or $668 \frac{4}{7}$ degrees for the 13th harmonic and the angle at which the e.m.f.s of this harmonic in each conductor will add is $668 \frac{4}{7} \div 9$ or

Design of a Sine Wave Generator

74.28°. The spread factor for this harmonic is equal to

$$\frac{2r \sin 25.72^\circ}{9 \times 2r \sin 37.14^\circ} = \frac{0.4339}{9 \times 0.603} = 0.0799$$

The effect of the 4/5 pitch on the 13th harmonic will be to cause the e.m.f.s of this harmonic in the two belts 144 degrees apart to add at $13 \times 36^\circ$ or 468° . This is equivalent to adding at $468-360$ or 108° . Hence the pitch factor for the 13th harmonic will be equal to the cosine of $1/2$ of 108° or 54° , which is 0.5878. In the two groups which are 60° apart the 13th harmonics will add at $13 \times 60^\circ$ or 780° . This is equivalent to adding at $780-720$ or 60 degrees. Hence the group factor for the 13th harmonic will be the same as for the fundamental, namely 1.732.

Therefore the value of the 13th harmonic e.m.f. in 360 electrical degrees of the

Design of a Sine Wave Generator

winding will be

$$18 e_{13} \times 0.0799 \times 0.5878 \times 1.732$$

$$= 1.464 e_{13}.$$

The equation for the 17th harmonic is

$$18 e_{17} \times (\text{spread factor})_{17} \times (\text{pitch factor})_{17} \times (\text{group factor})_{17}.$$

A belt will take up $17 \times 51 \frac{3}{7}$ or $874 \frac{2}{7}$ degrees for the 17th harmonic and the angle at which the e.m.f.s of this harmonic in each conductor will add is $874 \frac{2}{7} \div 9$ or 97.14° . The spread factor for this harmonic is then equal to

$$\frac{2r \sin 77.14^\circ}{9 \times 2r \sin 48.57^\circ} = \frac{0.9749}{9 \times 0.7497} = 0.1445.$$

The $4/5$ pitch will cause the 17th harmonics to add at 17×36 or 612 degrees which is equivalent to adding at $612-360$ or 252 degrees. The pitch factor for this harmonic is then equal to the cosine $1/2(360-$

Design of a Sine Wave Generator

252) or cosine 54° , which is 0.5878. In the two groups 60° apart the 17th harmonics will add at 17×60 or 1020° , which is equivalent to $(3 \times 360) - 1020$ or 60 degrees. The group factor for the 17th will therefore also be equal to 1.732.

The value of the 17th harmonic e.m.f. in 360 degrees of the winding will be

$$18 e_{17} \times 0.1445 \times 0.5878 \times 1.732$$

$$= 2.648 e_{17}.$$

Design of a Sine Wave Generator

Design of Winding.

If a winding is so designed that the coils have a throw of $4/5$ of the pole pitch this will eliminate the 5th harmonic and all multiples of the 5th. In this same winding the belts of conductors can be made to cover $51\frac{3}{7}$ degrees and the coils, which are 60° apart in phase, can be connected in series so that their e.m.f.s will add at 60° , thus eliminating the 7th and the 3rd harmonics and all multiples of the 7th and 3rd. A single layer winding of this type is shown in Fig. 8. The belts of conductors A, B, C, and D cover $51\frac{3}{7}$ degrees. Belts A and C are the two sides of the coil which has a pitch of 144° . The coils A-C and B-D are connected in series so that their e.m.f.s add at 60° . The front end of conductor 1 in belt A is connected to the front end of conductor 1 in belt C, the rear end of conductor 1 in belt C is connected to the rear end of conductor

Design of a Sine Wave Generator

2 in belt A, the front end of conductor 2 in belt A is connected to the front end of conductor 3 in belt C, etc.

Since all of these conductors are connected in series, it is immaterial in what order the conductors are connected into the circuit. Hence a coil can be made up of belts A and D with M as its center and another coil can be made up of belts B and C with P as a center. In order to avoid having the end connections cross over each other, conductor 1 in belt B can be connected on the front end to conductor 7 in belt C, the rear end of conductor 7 in belt C can be connected to the rear end of conductor 2 in belt B, the front end of conductor 2 in belt B can be connected to the front end of conductor 6 in belt C, etc. In this way the coils can be made up flat and will not cross over each other.

Design of a Sine Wave Generator

Flux Distribution.

When the winding is concentrated, the wave shape of the e.m.f. induced in the conductors is, in general, identical with the wave shape of the flux at no load. When load is applied the result of the armature reaction is to distort the flux curve and hence the wave shape of e.m.f. is also distorted. In a distributed winding, the e.m.f.s in each conductor will have a wave shape similar to the flux distribution. The distortion of the flux shows the presence of higher harmonics. When the winding described above is used, the third, fifth, and seventh harmonics and all multiples of these, would be cancelled out in the e.m.f. wave. This would leave the 11th, 13th, 17th, 19th, 23rd, 29th, and 31st harmonics, which may be present in the flux curve, to appear in the e.m.f. wave. Harmonics above the 31st would probably be of

Design of a Sine Wave Generator

such values as to be negligible, and since the half waves of e.m.f. are symmetrical about the zero axis, no even harmonics could appear. In order to reduce the 11th, 13th, 17th, etc. harmonics to the minimum, it is desirable to have a flux curve that is practically a cosine wave.

Design of a Sine Wave Generator

Armature Reaction.

Armature reaction is caused by the magnetomotive force set up by the current in the armature conductors. As was pointed out above, the effect of armature reaction is to distort the field flux. Referring to Fig. 9a, when the armature conductors are in the position shown the direction of the armature reaction will be along the line OM. ωt is the angle between the center line of armature reaction and the center line of the poles. The effect of the armature reaction is greatest when the center line of armature reaction coincides with the center line of the poles. In this case $\omega t = 0$. When the armature conductors have turned thru an angle of 90 degrees from the first position the direction of the armature reaction will be at right angles to the field flux and the effect will be a minimum. When they have passed thru 180°

Design of a Sine Wave Generator

from the original position the direction of the armature reaction will again be along the center line of the poles and as the current in the armature conductors has reversed, the direction of the armature reaction will be the same as in the first position and the effect will be a maximum. This variation is shown in Fig. 3b. At 0 degrees the effect of armature reaction is to decrease the field flux by the amount m_a . When the armature has turned thru 90° the curve representing the armature reaction coincides with the field flux at "b". At 180° the armature reaction is again a maximum and $n-c$ is equal to $m-a$. Hence, during one cycle of the fundamental the armature reaction completes two cycles, and when the angle ωt is 90° the armature reaction is zero. The value of the armature^{reaction} can therefore be expressed by the equation

$$a \cos 2 \omega t$$

Design of A Sine Wave Generator

where "a" is a constant.

The value of the actual flux is then equal to

$$\phi = \phi_k (1 - a \cos 2\omega t)$$

where ϕ_k is the flux due to the field poles.

The e.m.f. induced in the conductor A, Fig. 9a, will then be equal to

$$e = k\phi \cos (\omega t - \alpha)$$

the angle α being the angle between the line of armature reaction for the phase belt and the conductor A. Substituting the above value for ϕ

$$e = k\phi_k (1 - a \cos 2\omega t) \cdot \cos(\omega t - \alpha).$$

We can then consider only that part of the equation which is due to pulsating flux, i.e.

$$e' = \cos 2\omega t \times \cos(\omega t - \alpha)$$

Expanding the second term

$$\begin{aligned} e' &= \cos 2\omega t (\cos \omega t \times \cos \alpha + \sin \omega t \times \sin \alpha) \\ &= \cos \alpha \times \cos 2\omega t \cos \omega t + \sin \alpha \cos 2\omega t \sin \omega t \end{aligned}$$

Design of a Sine Wave Generator

$$\begin{aligned}
 e' &= \frac{\cos \alpha}{2} (\cos 3\omega t + \cos \omega t) \\
 &\quad + \frac{\sin \alpha}{2} (\sin 3\omega t - \sin \omega t) \\
 &= \frac{\cos \alpha}{2} \cos \omega t - \frac{\sin \alpha}{2} \sin \omega t \\
 &\quad + \frac{\cos \alpha}{2} \cos 3\omega t + \frac{\sin \alpha}{2} \sin 3\omega t
 \end{aligned}$$

Taking the 3rd harmonic component

$$\begin{aligned}
 e'' &= \frac{\cos \alpha}{2} \cos 3\omega t + \frac{\sin \alpha}{2} \sin 3\omega t \\
 &= \sqrt{\frac{\cos^2 \alpha + \sin^2 \alpha}{4}} \cdot \sin (3\omega t + \\
 &\quad \tan^{-1} \left[\frac{\cos}{\sin} \right]) \\
 &= \frac{1}{2} \sin (3\omega t + \tan^{-1} \cot \alpha) \\
 &= \frac{1}{2} \sin (3\omega t + \tan^{-1} \tan (90^\circ - \alpha)) \\
 &= \frac{1}{2} \sin (3\omega t + 90^\circ - \alpha) \\
 &= \frac{1}{2} \cos (3\omega t - \alpha)
 \end{aligned}$$



Design of a Sine Wave Generator

Hence a third harmonic will be introduced due to armature reaction. In the case of two conductors which are 60 degrees apart in phase the values of the e.m.f.s for the 3rd harmonics due to armature reaction will be

$$e_0'' = \frac{1}{2} \cos (3\omega t - \alpha)$$

and

$$e_{60}'' = \frac{1}{2} \cos (3\omega t - \alpha \pm 60^\circ).$$

These e.m.f.s would add at 60 degrees as shown in Fig. 10 and the resultant would be $\sqrt{3}$ times the 3rd harmonic. It is therefore evident that the 3rd harmonic which is caused by armature reaction, cannot be eliminated by connecting two groups, which are 60 degrees apart, in series.

Since there may be a third harmonic in the armature current this would also appear in the armature reaction and the pulsations of the flux due to this component will be of the sixth order. The flux, as affected by

Design of a Sine Wave Generator

this component, will then equal

$$\phi = \phi_k(1 - a \cos 5\omega t).$$

It can be shown in the same way as was shown above for the third harmonic, that this will cause the appearance of 5th and 7th harmonics in the e.m.f. wave form. In two belts of conductors which are 144° apart in phase the e.m.f.s add at $180-144$ or 36 degrees. The 5th harmonics in these belts would add at 36° as shown in Fig. 11. Hence the 5th harmonics which are the result of armature reaction will not be eliminated by a $4/5$ pitch. In a belt of conductors which cover $2/7$ of a pole pitch or $51\frac{3}{7}$ degrees, the e.m.f.s induced in the conductors will add together at the angle between the conductors. If there are 9 conductors in such a belt the e.m.f.s will add at $51\frac{3}{7} \div 9$ or $5\frac{5}{7}$ degrees. Fig. 12 shows a vector diagram of these e.m.f.s

Design of a Sine Wave Generator

for the 7th harmonics. These harmonics do not add up to zero and therefore the 7th harmonics which are induced by armature reaction are not eliminated by using a spread of $2/7$ of a pole pitch. Since the 3rd, 5th, and 7th harmonics are all present in the e.m.f. wave due to armature reaction and cannot be cancelled out by the methods discussed above, it is therefore necessary that the armature reaction be reduced to a minimum in order to make these harmonics negligible. The armature reaction can be diminished by using a long air gap and by placing short circuited windings in the pole faces.

Design of a Sine Wave Generator

Teeth.

Since the reluctance of the path of the flux directly underneath a tooth on the armature core is much less than the reluctance underneath a slot, the flux will have a tendency to move back and forth as the teeth pass beneath the pole shoe. The magnetic path in the air gap has practically a constant reluctance but a variable position. Hence the flux will be caused to swing to and fro. In Fig. 13a the position of the armature core is such that the slot "s" is in the center of the pole shoe and hence the flux at the center will be a minimum. As the armature moves to the right the tooth "m" assumes the position which was occupied by slot "s" and the flux at the center of the pole shoe will be a maximum. When the armature has again moved thru half of a slot pitch the slot "r" will be in the center and the flux

Design of a Sine Wave Generator

will be a minimum again. Hence while the armature moves thru one slot pitch the flux changes from a minimum to a maximum and back to a minimum, or thru one complete cycle. The frequency of these pulsations (referred to the fundamental) will be equal to the number of slot-pitches per pair of poles or will be equal to $2n$, where n is the number of slot pitches per pole pitch. These pulsations would tend to set up harmonics in the e.m.f. wave of the frequency $(2n \pm 1)$. This is shown in the developement of the theory of armature reaction. In order to use a winding scheme of $4/5$ pitch and $2/7$ spread, 35 slots per pole per phase would be necessary. This would mean a total of 105 slots per pole. Even tho a fractional pitch winding could be found which would permit using but one-fifth of this number of slots per pole, the width of the slots

Design of a Sine Wave Generator

would be too narrow to allow a large enough conductor to be used. If this arrangement could be used harmonics would be introduced of the order $2 \times (21 \pm 1)$ or 41 and 43. In order to prevent the introduction of these harmonics and the limiting of the capacity by the use of narrow slots, it is desirable to use a smooth core.

Design of a Sine Wave Generator

Part II - Design and Calculations.

Armature Coils.

From the above theoretical considerations a design was made which provided for the connecting of groups of coils 60 degrees apart in phase with a pitch of $\frac{4}{5}$ of the pole pitch and having a spread of conductors of $\frac{2}{7}$ of a pole pitch. The calculations and details of this design are described below.

In order that the capacity of the generator might be as large as possible, the largest possible size of armature conductors was selected. The size of these conductors was limited, however, because the larger the conductor the longer the air gap required (since a smooth core was used) and the fewer the conductors that could be placed in the allowable space. The greatest maximum ampere-turns of flux which could be obtained from the fields on the machine

Design of a Sine Wave Generator

which was to be re-designed was 3436.

Several sizes of d.c.c. wire were chosen and calculations made and the number of ampere-turns of flux necessary was determined. The largest practical size of round wire was found to be No. 6 Brown and Sharpe gage d.c.c. which is 174 mils in diameter outside the cotton covering. The method of calculation and results are shown below.

The gap between the iron of the armature core and the pole shoe was calculated by adding together the thicknesses of the various materials as follows:

- .010" - 2 thicknesses of varnished cloth on iron core
- .174" - diameter of No. 6 d.c.c. wire
- .010" - one thickness of micanite on armature conductors
- .032" - diameter of No. 20 binding wire
- .063" - assumed air gap
- .289" - total air gap

Design of a Sine Wave Generator

With No. 6 wire the number of wires per group for this machine was found to be 9. The total spread of the group will then be 9 times the diameter of the wire or $9 \times 0.174 = 1.566''$. In order to cover $2/7$ of a pole pitch the group will have to cover $51 \frac{3}{7}$ electrical degrees or, since this machine is a 10-pole machine, the group will cover $10 \frac{2}{7}$ mechanical degrees. Therefore the diameter of the armature to the centers of the conductors was found from the equation

$$\frac{1.566}{\pi} = \frac{10 \frac{2}{7}}{360}$$

Whence

$$D = 17.45''.$$

The diameter of the armature core will then be equal to

$$17.45 - (2 \times .97) = 17.256''.$$

The general e.m.f. equation for a single coil of Z turns is

Design of a Sine Wave Generator

$$E = 2.22 Z \phi f \times 10^{-8} \times f_c \times f_w$$

where ϕ is the flux per pole, f is the frequency, f_w , is the spread factor and f_c is the pitch factor. In this machine the spread of the conductors is to be made $2/7$ of the pole pitch in order to eliminate the 7th harmonic due to the flux distribution, and the pitch of the coils is to be $4/5$ of a pole pitch in order to eliminate the 5th harmonics. Hence the spread factor in this case will be the ratio between the vector sum of the e.m.f.s in each conductor and the arithmetic sum of these e.m.f.s. In Fig. 14 the angle θ_1 subtends the arc of the chord representing the e.m.f. in each conductor, and θ subtends the arc of the chord which is the vector sum of the e.m.f.s in the 9 conductors.

The spread factor will therefore be

$$\frac{2r \sin \frac{\theta}{2}}{9 \times 2r \sin \frac{\theta}{2}}$$

Design of a Sine Wave Generator

Since $\theta = 51 \frac{3}{7}$ degrees and $\theta_1 = 5 \frac{5}{7}$ degrees

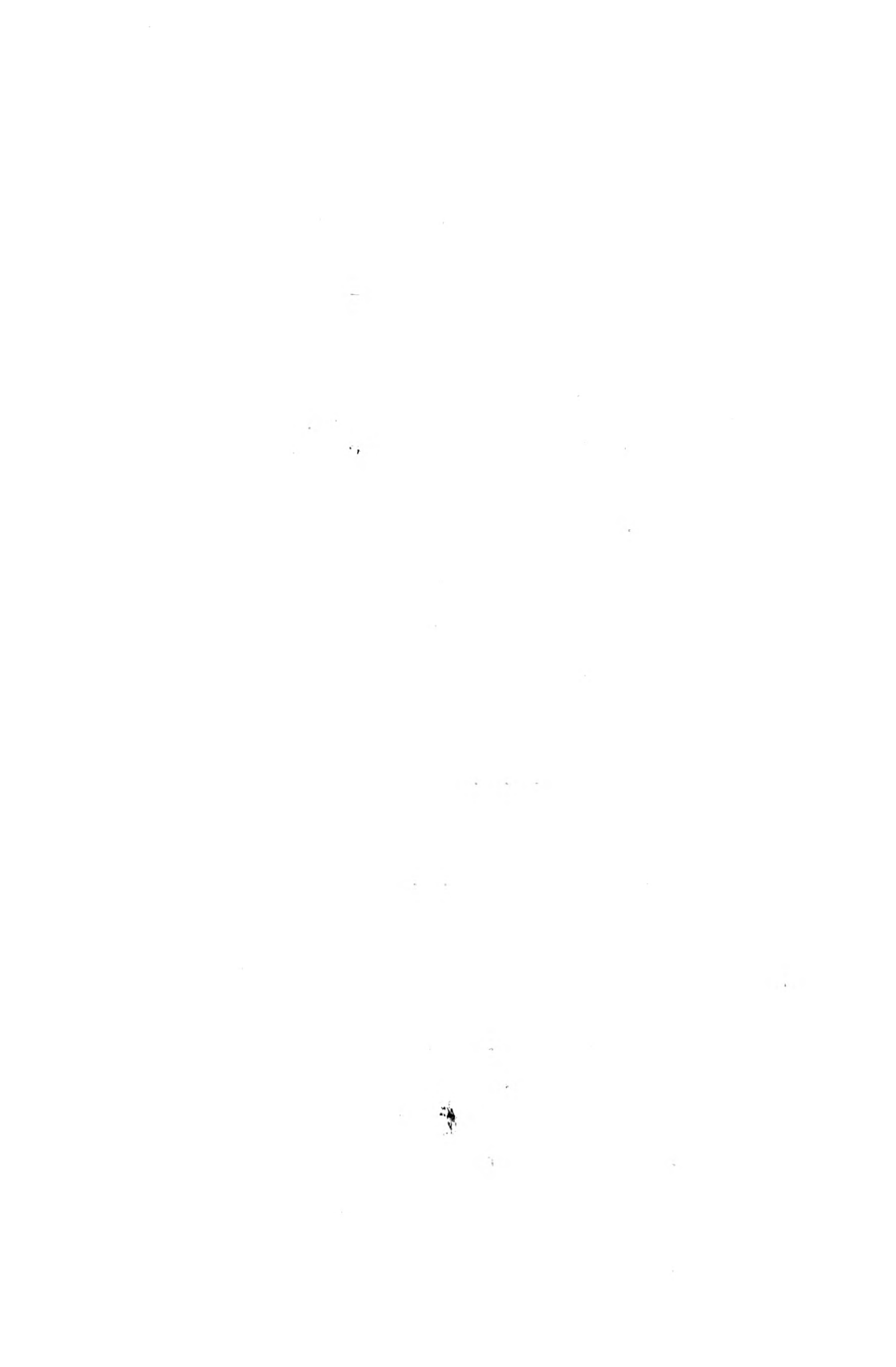
$$f_w = \frac{2r \sin 25 \frac{5}{7}}{18r \sin 2 \frac{6}{7}} = \frac{2r \times .4332}{18r \times .04972}$$

$$= 0.9697.$$

In order to obtain the pitch factor the ratio of the vector sum of the c.m.f. induced in each belt of conductors to the arithmetic sum is computed. In Fig. 15 a and b are the e.m.f.s induced in the two belts, and the vector c is the result of the addition of these e.m.f.s when the belts are 144° apart in phase. The pitch factor then will be

$$\frac{a \cos 18^\circ}{a} = 0.951.$$

The above value for E gives the volts per coil. There are to be two coils per pair of poles which are 60 degrees apart in phase, and there will be 5 pairs of



Design of a Sine Wave Generator

these coils in the complete winding. Since these coils are all connected in series, the total voltage at the machine terminals will be equal to

$$5 \times \sqrt{3} \times E$$

or if E_1 = total voltage,

$$\begin{aligned} E_1 &= 1.73 \times 5 \times 2.22 \times Z \times \phi \times f \times 10^{-8} \times f_c \times f_w \\ &= 19.2 \times Z \times \phi \times f \times 10^{-8} \times f_c \times f_w. \end{aligned}$$

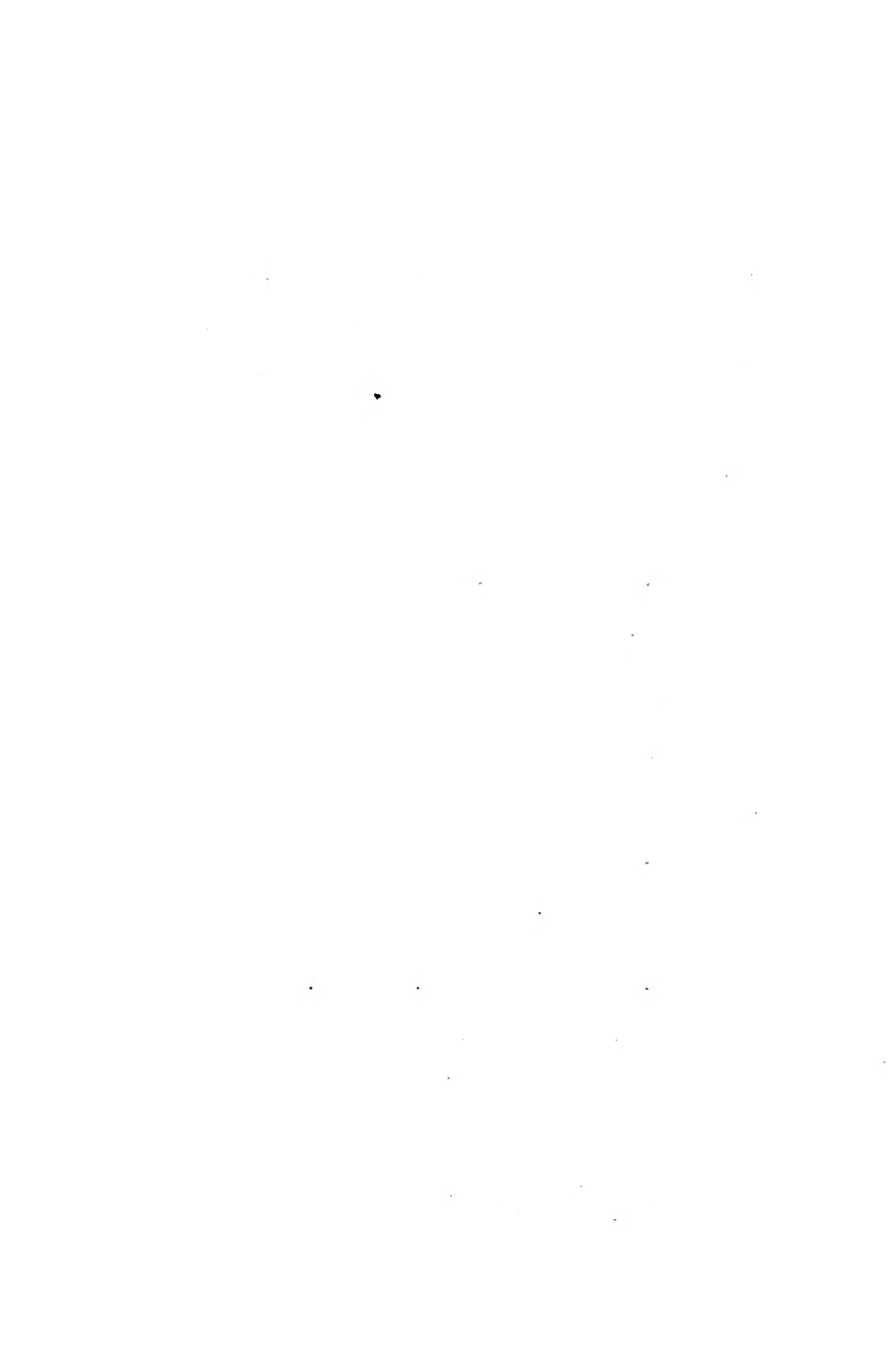
It is desired to obtain 125 volts from the machine at no load and at 60 cycles. Substituting these values of E_1 and f

$$\begin{aligned} 125 &= 19.2 \times 18 \times \phi \times 60 \times 10^{-8} \times .951 \times \\ &\quad .9697 \end{aligned}$$

$$\begin{aligned} \phi &= \frac{125 \times 10^8}{19.2 \times 18 \times 60 \times .951 \times .9697} \\ &= 654,000 \text{ lines.} \end{aligned}$$

The ampere-turns required on the field poles will be

$$IT = \frac{\phi \times \text{air gap in inches}}{3.2 \times \text{area of pole}}$$



Design of a Sine Wave Generator

The minimum air gap is 0.289" as computed above. The pole bore will be equal to the diameter of the armature core plus twice the total air gap.

$$17.256 + 2(0.289) = 17.834"$$

The pole pitch will be one-tenth of $\pi \times 17.834$ or 5.6" since the machine is a ten-pole machine. The width of the pole measured along the shaft is six inches. Hence the area of the pole shoe will be 5.6 x 6 or 33.6 square inches. Substituting these values in the above equation the number of ampere-turns will be

$$IT = \frac{654,000 \times 0.289}{3.2 \times 33.6} = 1758.$$

These calculations have been made on the assumption of a rectangular distribution of flux. In this design a sine distribution of flux is desired and hence the number of ampere-turns will have to be increased in the ratio of the area under a rectangle to the

Design of a Sine Wave Generator

area under a sine wave with the maximum height equal to the height of the rectangle and having the same base. This ratio is $\frac{\pi}{2}$.

The actual ampere-turns required will then be

$$\frac{\pi}{2} \times 1758 = 2760.$$

In calculating the capacity of the machine the circular mils per ampere was assumed to be 560. The total cross-section of No. 6 wire in circular mils is 26,250. Hence the capacity of the machine will be equal to $26,250 \div 560$ or 46.9 amperes. With a terminal voltage of 110 volts the K.V.A. capacity will be

$$\frac{110 \times 46.9}{1000} = 5.16 \text{ KVA.}$$

Layout of winding

The conductors were laid out in groups of

Design of a Sine Wave Generator

nine. These groups were then placed in pairs having the two groups in each pair 60 electrical degrees apart. The pairs were placed 144 electrical degrees apart and coils were made up whose sides constituted one group in each pair. The two different sizes of coils thus formed are shown in Plate II. The spacers required for the centers of these coils are shown in Plate I. The spacer C is placed between the coils on the armature core. The layout of the winding is shown in the bottom view on Plate I. A cross-sectional view of the winding in place on the core is shown on Plate IV. This plate also shows a section of the core.

The laminations for the core were cut from No. 26 gage iron which is .018" in thickness. The two end plates which were on the old armature core were used to hold the new core together and these plates were

Design of a Sine Wave Generator

bolted together by means of ten $9/16$ " bolts equally spaced around a circle of $10\ 3/4$ " diameter. The inside of the core was made $9\ 1/2$ " diameter.

Design of a Sine Wave Generator

Binding Wires

The armature coils are held in place by two bands of 31 turns each of No. 20 B and S phosphor bronze wire. The calculations for the stresses in these bands due to the centrifugal forces is shown below.

The total weight of the copper, spacers and insulation was calculated to be about 18.5 lb. From Kent's Handbook the formula for the centrifugal force in a rim is

$$F = .000341 W R r^2$$

where W is the weight of the rim in pounds, R is the mean radius in feet and r is the revolutions per minute. Substituting the various values in the above formula

$$F = .000341 \times 18.5 \times \frac{8.72}{12} \times (720)^2$$

$$= 2310 \text{ pounds.}$$

Since the resultant of the radial forces taken at right angles to the diameter is

$\frac{2}{\pi}$ times the sum of these forces, the total

Design of a Sine Wave Generator

force F is to be divided by $2 \times 2 \times 1.5708$ or 6.2832 to obtain the tensile stress on the cross-section of the band.

$$\frac{2310}{6.2832} = 368 \text{ pounds} = \text{stress on band.}$$

The breaking strength of one No. 20 phosphor bronze wire is 161 pounds. Since there are two bands of 31 wires each the total breaking strength of the bands will be

$$2 \times 31 \times 161 = 9982 \text{ pounds.}$$

The factor of safety then will be

$$\frac{9982}{368} = 27.1.$$

Design of a Sine Wave Generator

Design of Field System

Field Winding

The old field winding consisted of 10 layers of No. 15 wire with 60 turns per layer. This was covered with a series winding of 3 layers of No. 9 wire with 36 turns per layer. In the new machine there is no necessity for a series winding and hence the space occupied by this winding can be filled up with more of the shunt winding. A calculation was made for the field coil, assuming the No. 9 wire to be replaced with 6 layers of No. 15 wire, and the ten field coils connected in series across 110 volt circuit. The ampere-turns in this case were found to be 1908. This is entirely too low, as 2760 ampere-turns will be required for the air gap alone. A number of calculations were made with various sizes of wires and with the field coils connected in series-parallel,

Design of a Sine Wave Generator

five coils being in series across 110 volts. The greatest possible ampere-turns were obtained when 17 layers of No. 15 D.C.C. wire were used. This required 7 additional layers of No. 15 D.C.C. to be added to the old winding. The calculation of the ampere-turns for this field coil is shown below.

The length of the average turn was found to be 21.5 inches, and the resistance per 1000 feet of No. 15 wire is 3.56 ohms at 50°Cent. Since there were 17 layers and 60 turns per layer the total number of turns was $17 \times 60 = 1020$.

The total resistance of one coil is equal to

$$1020 \times \frac{21.5}{12} \times \frac{3.56}{1000} = 6.49 \text{ ohms.}$$

With 5 coils in series on a 110 volt circuit the voltage across each coil is

$$110 \div 5 = 22 \text{ volts.}$$

Design of a Sine Wave Generator

The maximum current will then be 22 ± 6.49
 $= 3.39$ amperes, and the ampere-turns will
 be $1020 \times 3.39 = 3460$.

The number of watts loss due to the resistance will be $W = I^2 R$, or

$$W = (3.39)^2 \times 6.49 = 74.5 \text{ watts.}$$

From the curve given on page 64 of Gray's "Electrical Machine Design" the allowable watts per square inch of external surface was found to be 0.8. The external area was computed to be 104.5 square inches, and the watts which can be radiated will be

$$0.8 \times 104.5 = 83.6$$

Hence the amount of heating for the field coil calculated above is well within the allowable limit.

As the design of the field coil gives 3460 ampere-turns and only 2760 are necessary for the air gap at 125 volts and no load, therefore 20 percent of the ampere-turns are

Design of a Sine Wave Generator

left to take care of the loss in the iron path of the magnetic circuit and the magnetic leakage. Armature reaction may also cut down the flux somewhat. This percentage of flux should be ample to compensate for all of these losses.

Design of a Sine Wave Generator

Pole Shoe

Since the flux is inversely proportional to the reluctance of the path, in order to obtain a cosine wave distribution of flux the air gap must vary inversely as a cosine wave. The equation governing this variation in the length of the gap is derived as follows:

In Fig. 16 the line A-B is the surface of the armature core and C-D is the outline of the pole shoe. In order to vary as a cosine function, the flux at any point at a distance x from the center line, will be

$$B = B_m \cos x$$

where B_m is the maximum flux, which is at 0.

$$B = \frac{\phi}{A} \quad \text{and} \quad \phi = AB$$

where A = area.

$$\phi = \text{m.m.f.} \times P$$

$$d\phi = M \cdot dP$$

Design of a Sine Wave Generator

$$dP = \frac{d\phi}{M} = \frac{(dA) \cdot B}{M} = \frac{w \, dx \cdot B}{M}$$

w being the width of the pole shoe and dx being a small increment on the circumference of the armature.

$$dP = \frac{w \, B_m \cos x}{M} \cdot dx$$

$$\text{but } dP = \frac{A}{l} = \frac{w \, dx}{y}$$

$$\text{Therefore } \frac{w \, dx}{y} = \frac{w \, B_m \cos x \, dx}{M}$$

$$\text{and } y = \frac{M}{B_m} \cdot \frac{1}{\cos x}$$

The minimum air gap as calculated above is 0.289". This will then be the air gap at point O, the center line of the pole shoe. Hence when $x = 0$, y will be equal to 0.289". Substituting these values in the above equation

$$.289" = \frac{M}{B_m}$$

since the cosine of 0° is 1.

Design of a Sine Wave Generator

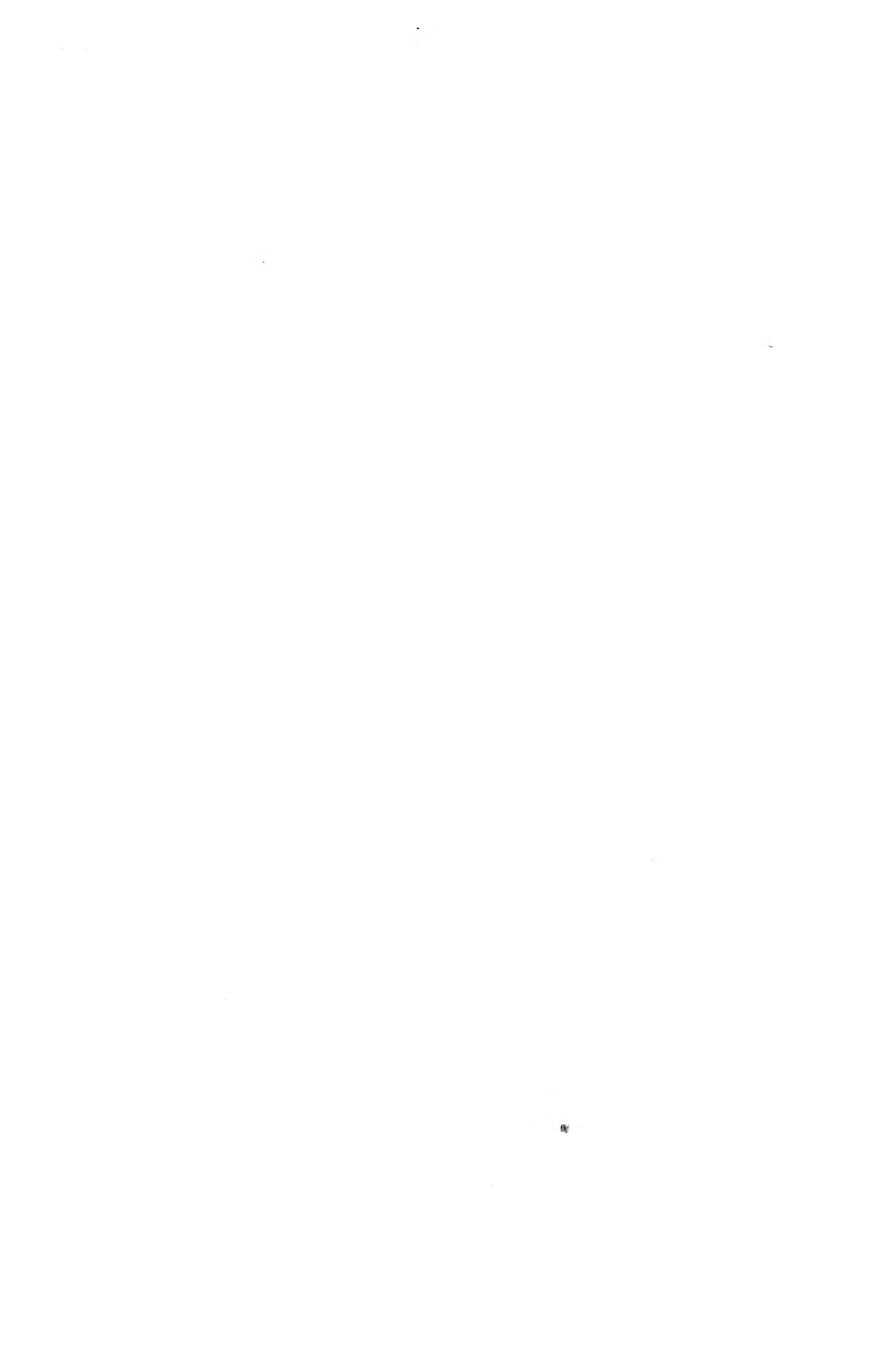
Replacing this value for the constant $\frac{M}{B_{in}}$, the equation becomes

$$y = 0.289 \frac{1}{\cos x}$$

By the use of this equation the length of the air gap was calculated for every $7\frac{1}{2}$ degrees, and this distance was laid off on a radial line passing thru the corresponding point on the armature circumference. Thru the points thus found a line was drawn giving the shape of the theoretical pole shoe. This construction is shown in Fig. 17. This theoretical shape is based upon the assumption that the lines of flux lie along the radial lines, which is not true. The paths of the flux were drawn in as curved lines which entered the pole shoe and the armature core tangent to perpendiculars at these points. The actual flux wave was then constructed by measuring the actual length

Design of a Sine Wave Generator

of these curved paths. The maximum flux would be at the center line of the pole and the flux would vary inversely as the length of the path. The resulting curve of flux is shown in Fig. 18 as curve A. Since the flux of one pole extends beyond the center line between the poles, the curve C was drawn in as representing the flux from the adjacent pole. This flux cannot actually exist in the air gap of the pole, but its effect would be to decrease the flux of the pole by this amount. Hence the ordinates of curve C were subtracted from curve A and the resulting curve B is the actual flux curve and strikes the zero axis at the center line between the poles which is at the point 12. The actual sine wave was drawn in, and is shown as a dotted line in Fig. 18. The curve B does not coincide with the sine curve due to the fact that the pole



Design of a Sine Wave Generator

shoe was not calculated by using the actual flux paths. The above method of drawing in the flux distribution curve is described in an article by Prof. Theo. Shou in the Electrical Review and Western Electrician for February 17, 1917, entitled "Calculations for Synchronous Machines".

From the theoretical pole shoe in Fig. 17 an actual pole shoe was drawn and is shown in Fig. 19. As will be noted from Fig. 17 the theoretical pole shoe is practically a straight line for about half the pole pitch. The actual pole shoe was therefore made as a straight line for about half the pole pitch in the middle part and the ends of the shoe were made flat and at an angle of $15\frac{1}{2}$ degrees to the middle portion. The probable paths of the flux are shown drawn in Fig. 19. The flux curve was also calculated and is shown as curve B in



Design of a Sine Wave Generator

Fig. 20. This curve deviates somewhat from a sine wave, but as the method of calculating the flux distribution is at best only approximate, it was thot that this shape of pole shoe was close enough to the theoretical shape for all practical purposes. The working drawing of the pole shoe is shown in Plate III. The field bore on the old machine was bored out to a diameter of $19 \frac{3}{8}$ ", and the pole shoes were held in place by means of two set screws which were screwed into the pole piece.

Analysis of Flux Distribution.

In order to determine the probable value of the harmonics which would be present in the flux curve, this curve (Curve B Fig. 20) was analyzed by the method described in Scientific Paper 205 of the Bureau of Standards. The equation thus obtained was

Design of a Sine Wave Generator

$$\begin{aligned}
 B = & 38.376 \sin \omega t - 1.80 \sin 3\omega t \\
 & - 0.482 \sin 5\omega t - 0.226 \sin 7\omega t \\
 & - 0.262 \sin 9\omega t - 0.213 \sin 11\omega t \\
 & - 0.107 \sin 13\omega t - 0.129 \sin 15\omega t \\
 & - 0.053 \sin 17\omega t.
 \end{aligned}$$

From this it will be seen that none of the harmonics is of a magnitude greater than 4.69 per cent of the fundamental. Since the 3rd, 5th, and 7th and their multiples will be eliminated by the winding, the 11th, 13th and 17th are of most interest.

We can re-write the above equation so that the value of the coefficient of the fundamental will be 100 and the coefficients of the harmonics will then be directly in percent. Also, since the 3rd, 5th and 7th harmonics and all multiples of them are eliminated by the winding, they can be omitted from the equation expressing the

Design of a Sine Wave Generator

e.m.f. of the machine. Assuming that the winding is a concentrated one with full pitch coils and that the groups of conductors add at 0 degrees, the equation for the e.m.f. will be

$$E = 100 \sin \omega t - 0.55 \sin 11\omega t \\ - 0.279 \sin 13\omega t - 0.138 \sin 17\omega t.$$

Since the winding is a distributed one and has a $4/5$ pitch and the groups add at 30° , the actual coefficients in the above equation will be reduced. The coefficients of the harmonics will be reduced as shown above in the discussion of spread factor.

The actual value of E , assuming the flux wave in Fig. 20 and not considering the effect of armature reaction, will be

$$E = 79.9 \sin \omega t - 0.095 \sin 11\omega t \\ - 0.0114 \sin 13\omega t - 0.0102 \sin 17\omega t.$$

Hence the value of the 11th harmonic, which is by far the largest in magnitude,

Design of a Sine Wave Generator

is only about 0.12 percent of the fundamental.

Design of a Sine Wave Generator

Conclusions.

By the application of the various devices of eliminating the harmonics in the e.m.f. wave of an alternator described in this report, an attempt has been made to design a machine that will generate a wave form which is close enough to a sine wave for all practical purposes of testing and experimentation. Some of the harmonics above the 9th, which are present in the flux distribution, will no doubt appear in the e.m.f. wave, but they will probably be negligible. There will also be some harmonics introduced by the pulsations of the armature reaction. However, the long air gap used will probably diminish the effect of armature reaction and thus reduce the harmonics from this cause to a negligible value. If the armature reaction is too pronounced, short-circuited turns imbedded in the pole shoe face may be necessary.



Design of a Sine Wave Generator

Appendix

Bibliography.

"Electromotive Force Wave-Shape in Alternators", Comfort A. Adams,

A.I.E.E. Proc. July, 1909.

"The Shape of the Pressure Wave in Electrical Machinery",

S.P.Smith and R.S.H.Boulding,

Jour. Inst. of E. E. January, 1915.

"Electrical Machine Design",

by Alexander Gray.

"Calculations for Synchronous Machines",

Theo. Shou,

Electrical Review and Western Electrician,

February 17, 1917.





THE
LIBRARY OF THE
MUSEUM OF
NATURAL HISTORY
AND
ZOOLOGY
OF THE
SMITHSONIAN INSTITUTION
WASHINGTON, D. C. 20560

